

# The Interdependence of Bitcoin and Financial Markets: A Copula-Garch Approach\*

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## Abstract

This paper aims to examine the relationship between Bitcoin and preminent financial indicators using Copula-GARCH method. In the study, we use closing prices of Bitcoin and US 10-Year Bond Yield, Gold Spot US Dollar, US Dollar Index, S&P 500, FTSE 100 and NIKKEI 225. To our knowledge, our paper is the first to examine this issue empirically. Analysis results show that there is no strong interdependence between Bitcoin and preminent financial indicators. These findings provide new information that will benefit policy makers, banks, financial investors, and risk managers in trading activities for both long-term and short-term strategies.

**Keywords:** Bitcoin; Copula-GARCH; Financial Markets.

JEL Classification: G1, G15, E44, C14, C22

## Finansal Piyasalar ve Bitcoin Bağımlılığı: Copula-Garch Yaklaşımı

### Öz

Bu makale, Bitcoin ile kritik finansal göstergeler arasındaki ilişkiyi Copula-GARCH yöntemini kullanarak incelemeyi amaçlamaktadır. Araştırmada, Bitcoin ve ABD 10-Yıllık Tahvil Verim, Altın Piyasa, ABD Doları Endeksi, S&P 500, FTSE 100 ve NIKKEI 225'in kapanış fiyatları kullanılmaktadır. Bildiğimiz kadarıyla, bu konuyu ampirik olarak inceleyen ilk makale budur. Analiz sonuçları, Bitcoin ve önde gelen finansal göstergeler arasında güçlü bir karşılıklı bağımlılık olmadığını göstermektedir. Bu bulgular, hem uzun vadeli hem de kısa vadeli stratejilerde alım satım faaliyetlerinde politika yapıcılara, bankalara, finansal yatırımcılara ve risk yöneticilerine fayda sağlayacak yeni bilgiler sunmaktadır.

**Anahtar Kelimeler:** Bitcoin; Copula-GARCH; Finansal Piyasalar.

JEL Sınıflandırması: G1, G15, E44, C14, C22

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## Introduction

Bitcoin is a peer-to-peer version of electronic cash (Nakamoto, 2008). Classified as cryptocurrency, Bitcoin has become popular in recent years. The currency can be said to have an intriguing logic (Eyal and Sirer, 2014). Although the Bitcoin world is prospering, there are several threats for users with regard to legal status and possible government sanctions (Grinberg, 2012). Despite having detractors, Bitcoin achieved an important role (Barber et al., 2012) and became a unique type of asset class in the financial markets within the last five years.

Rather than being issued by central organization such as a government or bank, it is completely reliant on cryptography, and the whole process of minting, storing and transfer is carried out by network of users (Ron and Shamir, 2013). Bitcoin was not created or controlled by a central organization, but by process called “mining”, one of the key concepts in Bitcoin world. Valid transactions are compiled in blocks, then these and previously accepted blocks are added to the ledger. All transactions must take place in the network, called blockchain, thus preventing users from double spending (O’Dwyer and Malone, 2014).

A major problem with Bitcoin is the possibility of double-spending (Garay et al., 2015), and therefore delayed payment verification is required (Karame et al., 2012). To avoid the double spending problem, the system depends on digital signatures to confirm ownership, and a public history of transactions (Reid and Harrigan, 2013).

There are some important general assumptions with regard to Bitcoin, such as stakeholders must accept the rules and validity of transactions, and most importantly, it must be confirmed that Bitcoin has a value (Kroll et al., 2013).

Bitcoin is represented by a series of signals called transaction, which have several inputs and outputs (Bonneau et al., 2015) and established on a transaction registry dispersed across all participants (Böhme et al., 2015). Hence, this is a Proof-of-Work-based currency, in that users themselves can create crypto coin, requiring a heavy computational burden.

In this paper, we used the copula approach to describe the dependence structure of variables of interest. Sklar (1959) first introduced the copula theory to allow flexible description of the dependence between variables. Nelsen (1999) provided a thorough description of copulas from a mathematic perspective. The copula function is powerful since it states that the multivariate distribution function can be decomposed into marginal

variables, and a density function copula, which completely describes the dependence framework of the variables. Embrechts et al. (2002) first employed the copula in the area of finance, and since has been widely applied in the field of financial risk management and portfolio decision problems. Cherubini et al. (2004) made a seminal contribution to the advent of pricing multivariate option by using copula. Mitchell et al. (2006) proposed Copula-GARCH models, which introduced the dynamic copula period. Patton (2006) reviewed the application of copula in financial time series. Bollerslev (2009) supplied references, leading to the extensive list of ARCH acronyms used in the literature. Mitchell and McKenzie (2003) established model selection criteria with the ability to correctly identify the data generating process in simulated data. Brooks and Burke (2003) reproduced a group of appropriately adjusted information criteria for selection of models from the AR-GARCH family. Du and Lai (2017) examine the dependence between electricity spot markets in core European countries including France, Germany, Austria and Switzerland based on copula models. Of the ten different copulas with both time invariant and varying parameters currently in use, the empirical results show that time-varying Student-t copula is the best model for the sample data. Albulescu et al. (2018) explores the bivariate dependence structure between the US Dollar and four major currencies (EUR, GBP, CAD, JPY) using daily data for the time-span 1999–2014, and utilize different time-invariant and time-varying copula functions with different forms of tail dependence, and find a positive dependence between all exchange rates.

We also investigated the volatility effect US 10-Year Bond Yield, Gold Spot US Dollar, US Dollar Index, S&P 500, FTSE 100 and NIKKEI 225 stock indices. Volatility of each stock market are modeled based on the multivariate GARCH(p,q), EGARCH, GJR-GARCH, PGARCH, and CGARCH models. We employ a two-step Copula-GARCH model to examine the dependence structure of daily stock markets returns. Firstly, we filter log-return daily data using univariate EGARCH, GJR-GARCH and PGARCH models to obtain standard residuals and construct the marginal distributions. Secondly, copulas are selected to join the estimated marginal distributions. The Akaike information criteria (AIC) and Schwartz information criteria (SIC) methods are then used to determine which copula provides best fitness to the market data.

Although many empirical studies have been conducted in the literature about Bitcoin, these studies are mostly based on Bitcoin price estimation, (Munim et al., 2019; McNally et al., 2018; Pant et al., 2018; Azari, 2019; Urquhart, 2017), return and volatility analysis (Dyhrberg, 2016; Katsiampa, 2017; Symitsi ve Chalvatzis, 2018; Ardia et al., 2019; Balcilar et al., 2017;

Lahmiri et al., 2018; Chaim ve Laurini, 2018; Katsiampa, 2018) and its use as a hedging instrument against other financial assets. (Dyhrberg, 2016; Bouri et al., 2017a; Bouri et al., 2017b; Urquhart ve Zhang, 2019; Pal ve Mitra, 2019; Wu et al., 2019). This study aims to eliminate uncertainty in the market as the first study that analyzes both volatility and dependency between bitcoin and leading financial markets. We aim to provide better insights of the volatility of Bitcoin returns, its dependence structures to financial markets in recent years. This paper will be a deeper extension to current literature in Bitcoin volatility modeling and forecasting with the financial time series GARCH model and different variations.

The main research theme of this study is to select a model capable of supporting our efforts to determine whether there is a connection between Bitcoin and preeminent financial markets. Employing such a model will provide an opportunity to reduce market uncertainty, and hence make a modest contribution to the current literature.

The structure of this paper is as follows. The second section presents literature review. The third and fourth sections discuss the model and the data, consecutively. In the fifth section, the empirical results are analysed. The last section provides final remarks.

## Literature Review

After its creation, much research followed on Bitcoin, generally conducted in the context of conceptual explanations, the introduction of cryptocurrency and the relationship between general economic indicators. In a study by Yermack (2015), Bitcoin was reviewed in terms of historical trading prices and it was described as acting more as an investment instrument than a currency, a finding supported by a similar study by Baur et al. (2015). Wijk (2013) used a statistical tool to establish a relationship between Bitcoin and the world's largest stock market indices (FTSE 100, Dow Jones, Nikkei 225), Dollar/Euro, Dollar/Yen and oil, to detect the short and long term effects of indicators on Bitcoin, and found that WTI oil price and Dollar/Euro exchange rates have long-term effects, and the Dow Jones index has short-term effect. Dyhrberg (2016a) studied the financial asset properties of Bitcoin by using the GARCH model. The author considered Bitcoin as a method of hedging, similar to gold or the dollar, and used the FTSE index, Dollar / Euro, Dollar / Pound exchange rate and federal fund rates to explain price volatility. In study by Dyhrberg (2016b), the asymmetric GARCH model was used to investigate the ability of Bitcoin to protect investors against market volatility and proposed that

Bitcoin could be used as a hedging tool against the US dollar in the short term, and against stocks in the FTSE index in the long term. Georgoula et al. (2015) attempted to identify the determinants of Bitcoin price, conducting a time series and sensitivity analysis which explored the short and long term relationships between Bitcoin price, basic economic variables, technological factors and tweets. Gronwald (2014) conducted a deeper analysis of Bitcoin price and behaviour using GARCH model to capture the more serious price movements that caused market shocks, showing that the model is very suitable for their proposed purpose and that the excessive price movements characterize the Bitcoin price. In a study by Bouri et al. (2016), a Dynamic Conditional Correlation Model was used to determine whether Bitcoin acted as a hedging tool and safe haven for large world stock indices, treasuries, oil, gold, general commodity index and US dollar index. The results demonstrate that Bitcoin is a weak protection tool, suitable only for diversification. However, it was found to have strong potential as a safe haven in one particular context, that is, against extreme weekly movements on Asian equities. Baek and Elbeck (2015) attempted to model the Bitcoin price using the S&P 500 index, the consumer price index, the Euro exchange rate and other economic indicators, but none of these economic variables were shown to affect the price. The authors reached the conclusion that Bitcoin is a purely speculative vehicle, with prices driven by investor intuition. Cheah and Fry (2015) pointed out that Bitcoin prices contain a substantial speculative component, and that Bitcoin markets are susceptible to bubbles. Examining the market efficiency of Bitcoin, Urquhart (2016) concluded that it does not currently have full efficiency, although further investigation found recent progress towards an efficient market. In another study, Urquhart (2017) reviewed Bitcoin price clustering, and found significant evidence of clustering at round numbers. A study by Nadarajah and Chu (2017) found that efficient market hypotheses are not valid for Bitcoin returns. Bariviera (2017) noted that daily returns exhibit persistent behaviour until 2014, after which the market became more informative. Katsiampa (2017), in the study of the volatility of Bitcoin returns, highlighted the importance of the AR-CGARCH model as the most appropriate for the inclusion of a long-running component of the short-term and conditional variance. Bouri et al. (2017) investigated the relationship between uncertainty and the Bitcoin market, revealing that Bitcoin acted as a hedge against uncertainty, a result echoed in a recent study by Demir et al. (2018). Yonghong et al. (2018) investigate time-dependent long-term memory in the Bitcoin market by using a rolling window approach and a new productivity index. Baur et al. (2018) find that Bitcoin exhibits distinctly

different return, volatility and correlation characteristics compared to other assets, including gold and US dollars. Holub and Johnson (2018) emphasizes that peer-to-peer (P2P) exchange plays an important role in global Bitcoin trade, while Dastgir et al. (2018) examines the causal relationship between Bitcoin (measured by Google Trends search queries) and Bitcoin returns in the period between January 2013 and December 2017.

## Model specification and estimation

### Copula Functions

The copula function is proposed to measure dependence of multivariate variables. Based on Sklar's well-known theorem (Sklar 1959), copulas allow the implementation of the division of the specification of a multivariate model into two parts: the marginal distributions on one side, the dependence structure (copula) on the other. Let  $X$  and  $Y$  be random variables with continuous distribution functions  $F_X$  and  $F_Y$ , which are uniformly distributed on the interval  $[0, 1]$ . Then, there is a copula such that for all  $x, y \in R$ ,

$$F_{XY}(X, Y) = C(F_X(X), F_Y(Y)) \quad (1)$$

The copula  $C$  for  $(X, Y)$  is the joint distribution function for the pair  $F_X(X), F_Y(Y)$  provided  $F$  and  $F_Y$  continuous.

The joint probability density of the variables  $X$  and  $Y$  is obtained from the copula density  $(u, v) = \frac{\partial^2 C(u, v)}{\partial u \partial v}$ , as follows:

$$f_{xy}(x, y) = c(u, v) f_x(x) f_y(y), \quad (2)$$

where  $f_x(x)$  and  $f_y(y)$  are the marginal densities of the random variables  $X$  and  $Y$ . According to Sklar (1959), an n-dimensional joint distribution can be decomposed into its n-univariate marginal distributions and an n-dimensional copula. In the extension of Sklar's theorem to continuous conditional distributions, Patton (2006) shows that the lower (left) and upper (right) tail dependence of two random variables is given for the copula as:

$$\lambda_l = \lim_{u \rightarrow 0} P(F_x(x) \leq u | F_y(x) \leq u) = \lim_{u \rightarrow 0} C(u, u)/u \tag{3}$$

$$\lambda_u = \lim_{u \rightarrow 1} P(F_x(x) > u | F_y(x) > u) = \lim_{u \rightarrow 1} 1 - 2u - C(u, u)/1 - u \tag{4}$$

where  $\lambda_l$  and  $\lambda_u \in [0, 1]$ .

### Copula Models

We introduce several copula models in this section (Nelsen, R. B. 1999); Gumbel copula, Clayton copula, Frank copula, Gaussian copula Student t copula, Survival Clayton Copula and Joe copula.

**Gumbel Copula:** This Archimedean copula is defined based on the generator function  $\phi(t) = (-\ln t)^\theta$ ,  $\theta \geq 1$ ;

$$C_\theta(u, v) = \exp\left(-[(-\ln u)^\theta + (-\ln v)^\theta]^{1/\theta}\right) \tag{5}$$

where  $\theta$  is the copula parameter restricted to. This copula is asymmetric, with more weight in the right tail. In addition, it is an extreme value copula.

**Clayton Copula:** This Archimedean copula is defined based on the generator function  $\phi(t) = \frac{t^{-\theta} - 1}{\theta}$ ,

$$C_\theta(u, v) = (u^{-\theta} + v^{-\theta} - 1). \tag{6}$$

where  $\theta$  is the copula parameter restricted to  $(0, \infty)$ . This copula is also asymmetric, but with more weight in the left tail.

**Frank Copula:** This Archimedean copula is defined based on the generator function:  $\phi(t) = -\ln \frac{e^{-\theta t} - 1}{e^{-\theta} - 1}$ ;

$$C_\theta(u, v) = -\frac{1}{\theta} \ln \left( 1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{(e^{-\theta} - 1)} \right) \tag{7}$$

where  $\theta$  is the copula parameter restricted to  $(0, \infty)$ .

**Gaussian copula:** The copula function can be written as:

$$C(u, v; \rho) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{2\rho rs - r^2 - s^2}{2(1-\rho^2)}\right) dr ds \quad (8)$$

where  $u = F_{Y_1}(y_1)$ ,  $v = F_{Y_2}(y_2)$  is the inverse of the standard normal distribution and  $\rho$  is the general correlation coefficient.

**Student-t copula:** The Student's-t copula allows for joint fat tails and an increased probability of joint extreme events compared with the Gaussian copula. This copula can be written as:

$$C_{\rho, \nu}(u, v) = \int_{-\infty}^{t_V^{-1}(u)} \int_{-\infty}^{t_V^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \left\{ 1 + \frac{x^2 - 2\rho xy + y^2}{\nu(1-\rho^2)} \right\}^{-(\nu+2)/2} ds dt \quad (9)$$

where  $\rho, \nu$  parameters of the t copula.

**Joe Copula:** This Archimedean copula is defined with based on the generator function:  $\phi(t) = -\ln[1 - (1-t)^\theta]$

$$C_\theta(u, v) = 1 - [(1-u)^\theta + (1-v)^\theta - (1-u)^\theta(1-v)^\theta]^{1/\theta} \quad (10)$$

where  $\theta$  is the copula parameter restricted to  $[1, \infty)$ .

**The BB8 (Frank-Joe):** Copula is

$$C(u_1, u_2, \theta, \delta) = \frac{1}{\delta} (1 - [1 - \frac{1}{1-(1-\delta)^\theta} (1 - (1-\delta u_1)^\theta)(1 - (1-\delta u_2)^\theta)]) \frac{1}{\theta} \quad (11)$$

with  $\theta \in [1, \infty) \cap \delta \in (0, 1]$ .

## Marginal Modelling

In order to build the model for bivariate distribution with the copula, the marginal distribution for the series must initially be formed. There are various models for commonly accepted financial time series returns. Engle and Bollerslev (1986) and Engle and Kroner (1995) propose ARCH and GARCH model, which have been widely applied to financial series. In their extensive review, Poon and Granger (2003) consider that important methodological viewpoints needed to be discussed, particularly regarding the evaluation of forecasts and classified volatility forecasts as belonging in one of the four

categories. There are a number of GARCH models; in this study, we combine ARMA (m,n) and GARCH (p,q), EGARCH, GJR-GARCH (p,q), PGARCH and CGARCH models for modeling daily financial returns, respectively. These models' specifications are as follows:

$$r_t = \lambda_0 + \sum_{j=1}^m \lambda_j r_{t-j} + \varepsilon_t - \sum_{i=1}^n \theta_i \varepsilon_{t-i} \tag{12}$$

$$r_t = w_0 + \sum_{i=1}^q \alpha_i u_{t-1}^2 + \sum_{j=1}^p \beta_j u_{t-1}^2 \tag{13}$$

$$\log(r_t) = w_0 + \sum_{i=1}^q \alpha_i \frac{|u_{t-i}|}{\sqrt{r_{t-i}}} + \sum_{i=1}^q \gamma_i \frac{u_{t-i}}{\sqrt{r_{t-i}}} + \sum_{j=1}^p \beta_j \log(u_{t-j}) \tag{14}$$

$$r_t = w_0 + \sum_{i=1}^p \beta_i r_{t-i} + \sum_{j=1}^q \alpha_j u_{t-j}^2 + \sum_{i=1}^q \gamma_j u_{t-j}^2 I_{t-j} \tag{15}$$

$$r_t^\delta = w_0 + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma \varepsilon_{t-i})^\delta + \sum_{i=1}^q \beta_j r_{t-j}^\delta \tag{16}$$

where m,n,p, q are positive integers , $u_t = \eta_t \sqrt{h_t}$  ,  $\eta_t \square f(0,1)$ , respectively  $\lambda_j$  ,  $\theta_i$  parameters of (AR) and (MA),  $w_0, \beta_i, \alpha_j, \gamma_j$  and  $\delta$  are ARCH(p,q), GARCH (1,1), EGARCH, GJR-GARCH (p,q) and PGARCH model parameters.

### Data

Daily Bitcoin (BTC) prices covers the period 07.08.2015-19.09.2018 and were downloaded from [www.coinmarketcap.com](http://www.coinmarketcap.com). For consistency, we eliminated weekend data due to the lack of corresponding data from other datasets. Bloomberg was the source of the other data (US 10-Year Bond Yield, Gold Spot US Dollar, US Dollar Index, S&P 500, FTSE 100 and NIKKEI 225). The observations, in total 787, reflect the daily prices between 07.08.2015-19.09.2018. Table 1 summarizes statistics of financial series and summarizes statistics of returns series, while Table 2 shows sizeable differences in the mean values for the seven markets, and also in the corresponding standard deviations. Skewness of returns out of Gold Spot is negative, indicating that

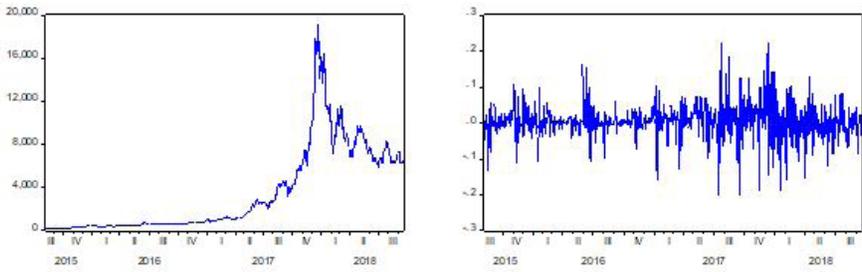
financial returns are skewed left, i.e. that the left tail is longer relative to the right. Gold Spot is skewed right. The high kurtosis of returns reveals that extreme value changes often occur when the tail of return distributions shows fatness. The Jarque-Bera (JB) test shows that the normality of each return series distribution is strongly rejected at 0.05 level, which means all price index distributions are non-normal. Finally, the Autoregressive Conditional Heteroscedasticity -Lagrange Multiplier (ARCH-LM) test indicates that strong ARCH effects exist in all financial return series. Graphical representations of the data employed are shown in Figures 1-7.

**Table 1.** Summary Statistics (Price Series)

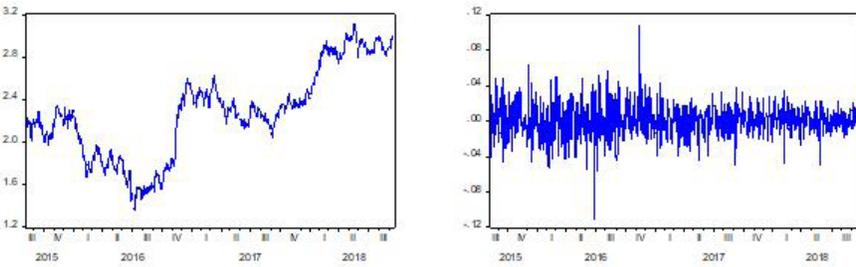
|                              | BTC      | US10-<br>Year<br>Bond<br>Yield | Gold<br>Spot | US<br>Dollar<br>Index | FTSE<br>100 | NIKKEI<br>225 | S&P 500  |
|------------------------------|----------|--------------------------------|--------------|-----------------------|-------------|---------------|----------|
| Mean                         | 3424,655 | 2,272382                       | 1,242523     | 95,72571              | 6,967637    | 19,49177      | 2,351210 |
| Median                       | 1058,840 | 2,273000                       | 1,254350     | 95,42000              | 7,176350    | 19,41537      | 2,347835 |
| Max                          | 19118,30 | 3,115000                       | 1,364900     | 103,2900              | 7,877450    | 24,12415      | 2,914040 |
| Min                          | 210,0700 | 1,358000                       | 1,050800     | 88,50000              | 5,536970    | 15,95202      | 1,829080 |
| Std. Dev.                    | 3993,569 | 0,422652                       | 0,074587     | 3,247193              | 0,577587    | 2,300957      | 0,291264 |
| Skewness                     | 1,388055 | -<br>0,000340                  | -0,700688    | 0,054261              | -0,504242   | 0,086467      | 0,224957 |
| Kurtosis                     | 4,398057 | 2,211989                       | 2,759570     | 2,587043              | 1,890523    | 1,881396      | 1,799128 |
| <u>Jarque</u><br><u>Bera</u> | 316,4092 | 20,33652                       | 66,20939     | 5,970654              | 73,62128    | 41,95866      | 53,85794 |
| Probability                  | 0,000000 | 0,000038                       | 0,000000     | 0,040523              | 0,000000    | 0,000001      | 0,000000 |

**Table 2.** Summary Statistics of return series

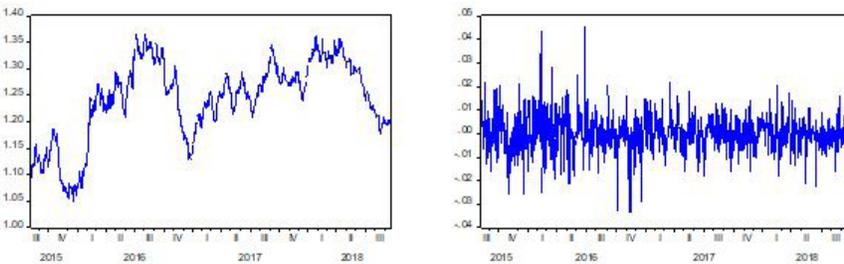
|                              | <b>Bitcoin</b> | <b>Year<br/>Bond<br/>Yield</b> | <b>Gold<br/>Spot</b> | <b>US<br/>Dollar<br/>Index</b> | <b>FTSE<br/>100</b> | <b>NIKKEI<br/>225</b> | <b>S&amp;P 500</b> |
|------------------------------|----------------|--------------------------------|----------------------|--------------------------------|---------------------|-----------------------|--------------------|
| Mean                         | 0,003986       | 0,000443                       | 0,000127             | -4,65e-050                     | 0,000111            | 0,000170              | 0.000428           |
| Median                       | 0,004305       | 0,000426                       | -7,66e-05            | 0,000000                       | 0,000323            | 0,000000              | 0.000493           |
| Max                          | 0,223513       | 0,107081                       | 0,045568             | 0,021577                       | 0,035150            | 0,074262              | 0.038291           |
| Min                          | -0,202077      | -0,110214                      | -0,033752            | -0,024185                      | -0,047795           | -0,082529             | -0.041843          |
| Std. Dev                     | 0,046821       | 0,019463                       | 0,008256             | 0,004446                       | 0,009078            | 0,013106              | 0.008120           |
| Skewness                     | -0,133615      | -0,092512                      | 0,234740             | -0,152676                      | -0,155529           | -0,215980             | -0.699039          |
| Kurtosis                     | 7,066050       | 5,649610                       | 6,177275             | 5,253610                       | 5,895575            | 9,778923              | 7.354931           |
| <u>Jarque</u><br><u>Bera</u> | 543,0948       | 230,7464                       | 337,4024             | 169,1675                       | 277,4030            | 1509,175              | 684.2597           |
| Probability                  | 0,00000        | 0,000000                       | 0,000000             | 0,000000                       | 0,000000            | 0,000000              | 0.000000           |
| ARCH LM                      | 37,04137       | 28,69334                       | 2,603280             | 6,049454                       | 112,1329            | 18,33321              | 86,78615           |
| Probability                  | 0,00000        | 0,00000                        | 0,1066               | 0,0139                         | 0,00000             | 0,00000               | 0,00000            |



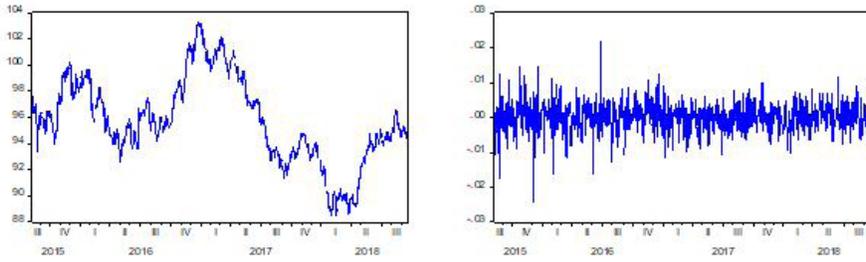
**Figure 1.** Change over years of BTC series and BTC return series



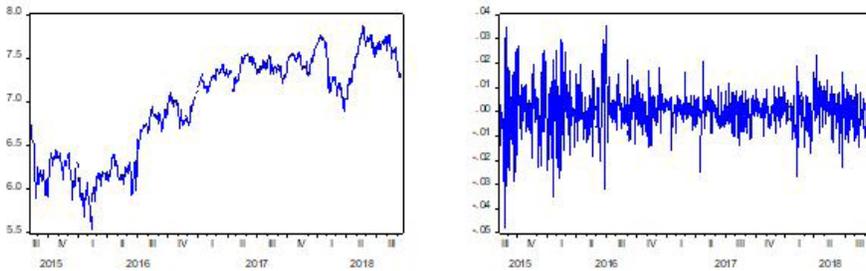
**Figure 2.** Change over years of US10-Year Bond Yield series and US10-Year Bond Yield return series



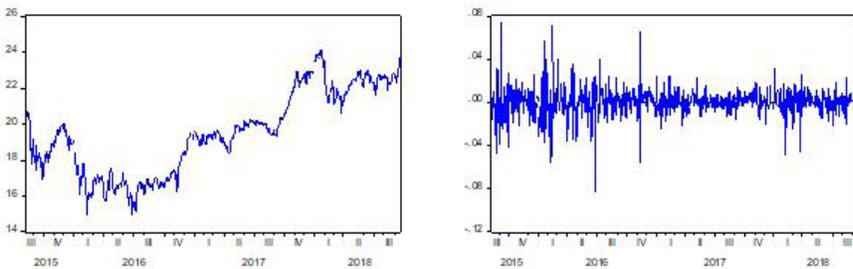
**Figure 3.** Change over years of Gold Spot series and Gold Spot return series



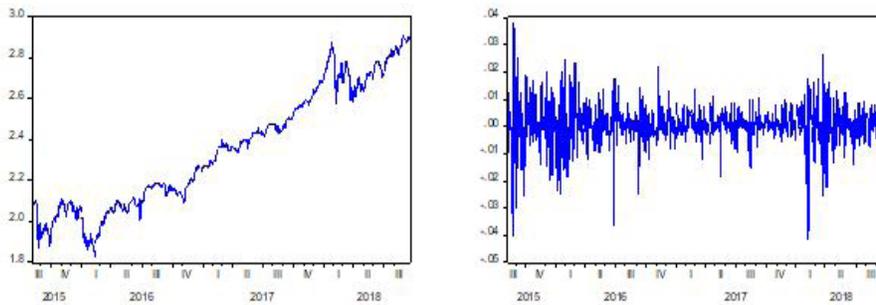
**Figure 4.** Change over years of US Dollar Index series and US Dollar Index return series



**Figure 5.** Change over years of FTSE 100 series and FTSE 100 Index return series



**Figure 6.** Change over years of NIKKEI 225 series and NIKKEI 225Index return series

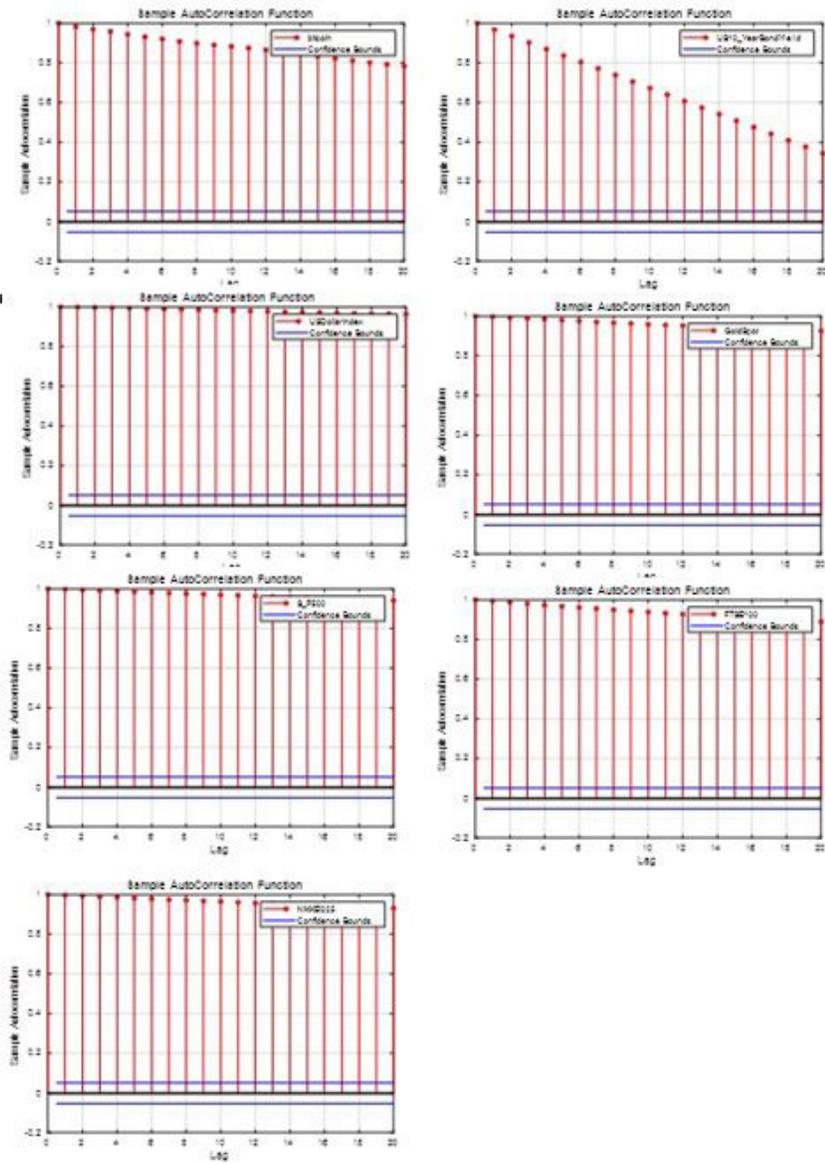


**Figure 7.** Change over years of S&P 500 series and S&P 500 Index return series

## Empirical Results

### Results of marginal distributions

We used the ARMA, GARCH, EGARCH, GJR-GARCH, PGARCH and CGARCH models for financial return series, and selected the most suitable model based on AIC and SIC model selection criteria. All the parameters estimate of marginal distributions are included in Table 3 and Table 4, which summarize the best fit model for all marginal distributions employed: the best models for the marginal; BTC, US 10-Year Bond Yield, Gold Spot, US Dollar Index, FTSE 100, Nikkei 225 and S&P 500 ARMA (0,0)-EGARCH(1,1,1) ARMA (3,4)- GJR-GARCH, ARMA (2,2)-CGARCH, ARMA (3,4)- GJR-GARCH, ARMA (4,0)- GJR-GARCH, ARMA (3,3)- CGARCH and ARMA (2,2)- PGARCH respectively. Based on the obtained results, Bitcoin and SP500 is modelled used PGARCH. In the PARCH model, from equation (17),  $\delta$  and  $\gamma$  parameters represent the power parameter of standard deviation and the asymmetric effect, respectively. From Table 4, for BTC  $\gamma$  parameter is negative and for SP500  $\gamma$  parameter is positive. US 10-Year Bond Yield, US Dollar Index and FTSE 100 are modelled via GJR- GARCH model. This model shows that good news and bad news might have different effects on volatility. The leverage effect is obtained as  $(\alpha + \gamma)$  of negative shocks which is larger than  $(\alpha)$  of positive shocks. In this model, if  $\gamma > 0$ , the leverage effect exists. As can be seen from Table 4, for US 10-Year Bond Yield, US Dollar Index and FTSE 100,  $\gamma$  parameter is positive, namely, this series has leverage effect and Gold Spot and Nikkei 225 are modelled via CGARCH. For US 10-Year Bond Yield, US Dollar Index, FTSE 100, Nikkei 225 and S&P 500, the results of ARCH-LM test show that neither autocorrelation nor ARCH effects exist in the residuals; however, for Bitcoin and Gold Spot series, it is seen that the variance problem and the ARCH effect are not completely removed (figure-8).w



**Figure 8.** For BTC-US10-Year Bond Yield, Gold Spot, US Dollar Index FTSE 100, NIKKEI 225, S&P 500 pairs Auto Correlation Function, respectively

**Table 3.** Mean Equation for marginal distribution model of financial series

|                      | Bitcoin  | US10-Year Bond Yield | Gold Spot   | US Dollar Index | FTSE 100  | NIKKEI 225  | S&P 500     |
|----------------------|----------|----------------------|-------------|-----------------|-----------|-------------|-------------|
| <b>Mean Equation</b> | <b>0</b> | <b>-3,4</b>          | <b>-2,2</b> | <b>-3,4</b>     | <b>-4</b> | <b>-3,3</b> | <b>-2,2</b> |
| $\lambda_1$          | -        | -0,99609             | -0,06296    | -0,39664        | -0,0008   | -1,17431    | 0,012023    |
| $\lambda_2$          | -        | 0,985021             | -0,96117    | 0,380491        | -0,03956  | 0,588045    | 0,925081    |
| $\lambda_3$          | -        | 0,984231             | -           | 0,981819        | 0,006898  | 0,779244    | -           |
| $\lambda_4$          | -        | -                    | -           | -               | -0,10636  | -           | -           |
| $\theta_1$           | -        | 0,954115             | 0,034028    | 0,418426        | -         | 1,157923    | -0,01662    |
| $\theta_2$           | -        | -1,01899             | 0,954018    | -0,39702        | -         | -0,66248    | -0,98338    |
| $\theta_3$           | -        | -0,95503             | -           | -1,00615        | -         | -0,83443    | -           |
| $\theta_4$           | -        | 0,01993              | -           | -0,01526        | -         | -           | -           |
| AIC                  | -3,27465 | -5,04828             | -6,32842    | -7,98112        | -6,55623  | -5,8264     | -6,78597    |
| SIC                  | -3,26277 | -4,99484             | -6,2928     | -7,92768        | -6,5206   | -5,7789     | -6,75034    |
| HQIC                 | -3,27008 | -5,02774             | -6,31473    | -7,96058        | -6,54253  | -5,80814    | -6,77227    |

**Table 4.** Variance Equation for marginal distribution model of financial series

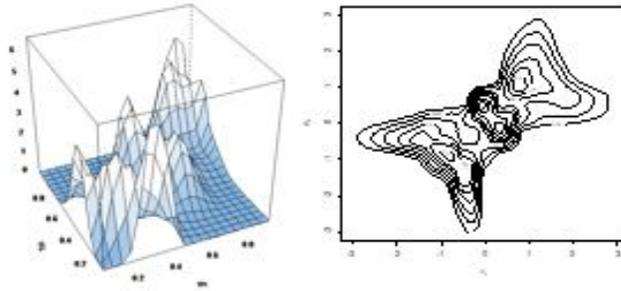
| Variance Equation | Bitcoin  |           | US10-Year Bond Yield |           | Gold Spot |           | US Dollar Index |           | FTSE 100 |           | NIKK EI 225 |           | S&P 500  |           |
|-------------------|----------|-----------|----------------------|-----------|-----------|-----------|-----------------|-----------|----------|-----------|-------------|-----------|----------|-----------|
| GARCH(1,1)        | Gauss an | Studen tt | Gauss an             | Studen tt | Gauss an  | Studen tt | Gauss an        | Studen tt | Gauss an | Studen tt | Gauss an    | Studen tt | Gauss an | Studen tt |
| $\omega_0$        | 4,84E-05 | 1,66E-05  | -2,15E-07            | -3,12E-08 | 7,20E-08  | 6,81E-05  | -4,72E-08       | -1,65E-08 | 4,13E-06 | 3,54E-06  | 4,72E-06    | 5,72E-06  | 4,08E-06 | 1,88E-06  |
| $\alpha$          | 0,13022  | 0,28607   | 0,01455              | 0,0154    | 0,01116   | 0,1500    | 0,01258         | 0,0066    | 0,13886  | 0,1556    | 0,14116     | 0,1500    | 0,21692  | 0,1912    |
| $\beta$           | 0,85955  | 0,83541   | 0,98437              | 0,9826    | 0,98674   | 0,6000    | 1,01527         | 1,0069    | 0,80392  | 0,8039    | 0,84036     | 0,8403    | 0,72188  | 0,7992    |
| AIC               | 3,52619  | 3,77873   | 5,15759              | 5,1843    | 6,81100   | 6,3886    | 8,06131         | 8,0620    | 6,82708  | 6,8668    | 6,10463     | 6,2336    | 7,14172  | 7,2406    |
| SIC               | 3,50241  | 3,74873   | 5,13382              | 5,1546    | 6,78723   | 6,2588    | 8,03754         | 8,0373    | 6,80331  | 6,8371    | 6,08085     | 6,2039    | 7,11795  | 7,2109    |
| ARCH LM           | 0,03135  | 0,24198   | 11,2436              | 10,325    | 6,16380   | 4,6678    | 1,06575         | 1,3157    | 0,63320  | 0,1912    | 0,35883     | 0,5208    | 2,12034  | 2,2209    |
| EGARCH(1,0,1)     | Gauss an | Studen tt | Gauss an             | Studen tt | Gauss an  | Studen tt | Gauss an        | Studen tt | Gauss an | Studen tt | Gauss an    | Studen tt | Gauss an | Studen tt |
| $\omega_0$        | 6,39951  | 4,89921   | 8,16087              | 8,1445    | 9,59487   | 9,3787    | 10,8716         | 10,880    | 9,91339  | 9,8015    | 8,89820     | 8,6986    | 10,1231  | 9,7748    |
| $\alpha$          | 0,33456  | 1,18444   | 0,32753              | 0,3049    | 0,01000   | 0,2849    | 0,04384         | 0,0481    | 0,52023  | 0,5103    | 0,23278     | 0,4117    | 0,50434  | 0,6961    |
| $\gamma$          | 0,06015  | 0,02416   | 0,06074              | 0,0165    | 0,01000   | 0,1379    | 0,08548         | 0,0901    | 0,06041  | 0,1692    | 0,18077     | 0,1788    | 0,09148  | 0,0674    |
| AIC               | 3,33015  | 3,59705   | 5,06430              | 5,1034    | 6,74594   | 6,8590    | 7,99037         | 8,0397    | 6,68440  | 6,7657    | 5,89419     | 6,1447    | 6,92464  | 7,0820    |
| SIC               | 3,30638  | 3,56734   | 5,40528              | 5,0937    | 6,72217   | 6,8293    | 7,96660         | 8,0099    | 6,66062  | 6,7359    | 5,87041     | 6,1149    | 6,90087  | 7,0522    |
| ARCH LM           | 0,27825  | 1,07649   | 0,24493              | 0,2890    | 2,76339   | 0,0145    | 2,31535         | 2,1050    | 1,02937  | 1,1115    | 0,19386     | 0,1122    | 0,61621  | 0,0737    |

|               | (0,5978)     | (0,2995)     | (0,6207)      | (0,5908)      | (0,0964)     | (0,9040)     | (0,1281)      | (0,1468)      | (0,3103)     | (0,2918)     | (0,6597)     | (0,7376)     | (0,4325)     | (0,7859)     |
|---------------|--------------|--------------|---------------|---------------|--------------|--------------|---------------|---------------|--------------|--------------|--------------|--------------|--------------|--------------|
| EGARCH(0,1,1) | Gauss        | Studen       | Gauss         | Studen        | Gauss        | Studen       | Gauss         | Studen        | Gauss        | Studen       | Gauss        | Studen       | Gauss        | Studen       |
| $\omega_0$    | 3,49087<br>0 | 2,73628<br>9 | 0,00171<br>1  | 0,0009<br>82  | 0,00234<br>2 | 0,6507<br>57 | 2,58311<br>1  | 2,4435<br>07  | 0,04905<br>7 | 0,0407<br>08 | 0,57436<br>9 | 0,5171<br>64 | 0,81207<br>2 | 0,2179<br>18 |
| $\gamma$      | 0,03138<br>4 | 0,13655<br>4 | 0,02484<br>5  | 0,0234<br>04  | 0,01989<br>1 | 0,0758<br>16 | 0,11160<br>3  | 0,1147<br>30  | 0,09594<br>8 | 0,1019<br>52 | 0,23064<br>6 | 0,2481<br>16 | 0,25198<br>9 | 0,2279<br>22 |
| $\beta$       | 0,43012<br>6 | 0,40261<br>9 | 1,00008<br>7  | 1,0002<br>03  | 1,00046<br>4 | 0,9322<br>14 | 0,76194<br>1  | 0,7749<br>30  | 0,99480<br>3 | 0,9959<br>59 | 0,93703<br>6 | 0,9430<br>43 | 0,91896<br>2 | 0,9789<br>13 |
| AIC           | 3,27801<br>4 | 3,55355<br>6 | 5,17663<br>2  | 5,1964<br>74  | 6,80768<br>9 | 6,8593<br>47 | 8,00055<br>6  | 8,0478<br>43  | 6,86312<br>2 | 6,8876<br>00 | 6,12808<br>6 | 6,2579<br>00 | 7,08677<br>8 | 7,1864<br>22 |
| SIC           | 3,26887<br>3 | 3,52383<br>8 | 5,15285<br>8  | 5,1667<br>56  | 6,78391<br>5 | 6,8296<br>29 | 7,96778<br>1  | 8,0181<br>26  | 6,84012<br>2 | 6,8578<br>82 | 6,10431<br>5 | 6,2281<br>82 | 7,06300<br>4 | 7,1567<br>04 |
| ARCHLM        | 40,6971<br>1 | 36,6554<br>0 | 13,0717<br>0  | 12,903<br>83  | 5,04751<br>4 | 2,6206<br>60 | 2,74770<br>1  | 2,6048<br>45  | 13,2530<br>4 | 11,525<br>40 | 2,08078<br>5 | 2,0653<br>03 | 22,9881<br>9 | 13,020<br>91 |
| EGARCH(1,1,1) | Gauss        | Studen       | Gauss         | Studen        | Gauss        | Studen       | Gauss         | Studen        | Gauss        | Studen       | Gauss        | Studen       | Gauss        | Studen       |
| $\omega_0$    | 0,54325<br>8 | 0,19635<br>1 | 0,00313<br>3  | 0,0025<br>11  | 9,84099<br>5 | 9,7643<br>28 | 2,45100<br>4  | 2,3305<br>23  | 0,48937<br>7 | 0,4442<br>81 | 0,72491<br>5 | 0,7169<br>93 | 0,88830<br>5 | 0,6360<br>24 |
| $\alpha$      | 0,28273<br>2 | 0,26179<br>7 | 0,00191<br>5  | 0,0147<br>87  | 0,26746<br>7 | 0,2912<br>96 | 0,04883<br>0  | 0,0481<br>93  | 0,13827<br>6 | 0,1463<br>00 | 0,17442<br>8 | 0,1773<br>66 | 0,21386<br>9 | 0,2273<br>72 |
| $\gamma$      | 0,00194<br>4 | 0,08398<br>5 | 0,02759<br>7  | 0,0313<br>16  | 0,07501<br>1 | 0,1338<br>84 | 0,10594<br>5  | 0,1083<br>70  | 0,12889<br>5 | 0,1424<br>83 | 0,19739<br>6 | 0,2353<br>28 | 0,21581<br>7 | 0,1935<br>23 |
| $\beta$       | 0,94503<br>1 | 0,99372<br>7 | 0,99974<br>0  | 0,9992<br>16  | 0,04403<br>0 | 0,0407<br>07 | 0,77758<br>4  | 0,7887<br>87  | 0,96031<br>8 | 0,9660<br>78 | 0,93420<br>6 | 0,9349<br>66 | 0,92756<br>8 | 0,9537<br>00 |
| AIC           | 3,52769<br>7 | 3,78887<br>6 | 5,17436<br>3  | 5,1951<br>85  | 6,76141<br>3 | 6,8565<br>87 | 7,99981<br>8  | 8,0466<br>55  | 6,86779<br>1 | 6,8962<br>12 | 6,17645<br>3 | 6,2869<br>47 | 7,18536<br>5 | 7,2676<br>92 |
| SIC           | 3,49798<br>0 | 3,75321<br>5 | 5,14464<br>2  | 5,1595<br>24  | 6,73196<br>9 | 6,8209<br>26 | 7,97010<br>1  | 8,0109<br>94  | 6,83819<br>5 | 6,8605<br>51 | 6,14673<br>5 | 6,2512<br>86 | 7,15564<br>8 | 7,2320<br>31 |
| ARCHLM        | 0,00174<br>4 | 0,01588<br>5 | 12,5810<br>0  | 11,647<br>70  | 0,01948<br>2 | 0,0053<br>32 | 0,62077<br>4  | 0,5759<br>52  | 0,74982<br>0 | 0,5840<br>06 | 0,03102<br>8 | 0,1505<br>35 | 2,24016<br>8 | 2,3235<br>16 |
| GJR-GARCH     | Gauss        | Studen       | Gauss         | Studen        | Gauss        | Studen       | Gauss         | Studen        | Gauss        | Studen       | Gauss        | Studen       | Gauss        | Studen       |
| $\omega_0$    | 4,49E-<br>05 | 7,54E-<br>06 | -3,14E-<br>07 | -4,14E-<br>07 | 1,75E-<br>07 | 5,44E-<br>07 | 5,67E-<br>10  | -3,51E-<br>09 | 2,93E-<br>06 | 2,98E-<br>06 | 9,55E-<br>06 | 9,28E-<br>06 | 4,23E-<br>06 | 2,57E-<br>06 |
| $\alpha$      | 0,13495<br>6 | 0,34975<br>3 | 0,01376<br>8  | 0,0119<br>18  | 0,01611<br>9 | 0,0255<br>06 | 0,00422<br>6  | 0,0050<br>77  | 0,02185<br>7 | 0,0185<br>03 | 0,05621<br>2 | 0,0565<br>54 | 0,05764<br>0 | 0,0297<br>97 |
| $\gamma$      | 0,02755<br>8 | 0,20790<br>4 | 0,01981<br>2  | 0,0166<br>30  | 0,02063<br>2 | 0,0217<br>97 | 0,00068<br>8  | 0,0001<br>23  | 0,21385<br>6 | 0,2438<br>34 | 0,46639<br>7 | 0,4956<br>91 | 0,29982<br>2 | 0,3275<br>96 |
| $\beta$       | 0,86874<br>1 | 0,86111<br>3 | 1,00219<br>9  | 1,0021<br>42  | 0,99003<br>7 | 0,9760<br>99 | 1,00368<br>6  | 1,0043<br>78  | 0,87119<br>6 | 0,8530<br>57 | 0,78729<br>4 | 0,7925<br>88 | 0,72737<br>1 | 0,7813<br>53 |
| AIC           | 3,52467<br>0 | 3,78773<br>9 | 5,18166<br>2  | 5,1962<br>01  | 6,81564<br>6 | 6,8792<br>24 | 8,04222<br>2  | 8,0626<br>45  | 6,87259<br>5 | 6,8995<br>36 | 6,17294<br>6 | 6,2894<br>41 | 7,17300<br>0 | 7,2666<br>02 |
| SIC           | 3,49495<br>2 | 3,75207<br>7 | 5,15194<br>4  | 5,1605<br>40  | 6,78592<br>8 | 6,8435<br>63 | 8,01250<br>5  | 8,0269<br>84  | 6,84287<br>7 | 6,8688<br>75 | 6,14322<br>8 | 6,2537<br>80 | 7,14328<br>3 | 7,2309<br>41 |
| ARCHLM        | 0,06446<br>1 | 0,08557<br>0 | 11,5697<br>2  | 11,686<br>22  | 5,92201<br>2 | 5,7918<br>55 | 1,12373<br>6  | 1,2853<br>96  | 0,09479<br>9 | 0,0095<br>68 | 0,43310<br>3 | 0,6266<br>81 | 1,03451<br>8 | 1,0026<br>12 |
| PGARCH        | Gauss        | Studen       | Gauss         | Studen        | Gauss        | Studen       | Gauss         | Studen        | Gauss        | Studen       | Gauss        | Studen       | Gauss        | Studen       |
| $\omega_0$    | 0,00025<br>3 | 0,00026<br>8 | -7,55E-<br>05 | -1,57E-<br>06 | 0,00625<br>6 | 0,0068<br>83 | -8,65E-<br>09 | 8,46E-<br>09  | 7,23E-<br>05 | 6,15E-<br>05 | 0,00146<br>6 | 0,0017<br>19 | 0,00278<br>5 | 0,0004<br>29 |
| $\alpha$      | 0,13396<br>6 | 0,16364<br>9 | 0,00707<br>6  | 0,0064<br>53  | 0,12374<br>2 | 0,0775<br>44 | 0,00247<br>9  | 0,0022<br>42  | 0,07315<br>9 | 0,0808<br>86 | 0,12298<br>4 | 0,1319<br>68 | 0,14048<br>4 | 0,1352<br>14 |
| $\gamma$      | 0,05152<br>2 | 0,36760<br>3 | 0,99926<br>1  | 0,9991<br>36  | 0,22420<br>3 | 0,9986<br>12 | 0,94449<br>6  | 0,9260<br>12  | 0,99988<br>4 | 0,9999<br>37 | 0,99994<br>3 | 0,9998<br>41 | 0,99671<br>8 | 0,9911<br>82 |
| $\beta$       | 0,86847<br>4 | 0,90184<br>1 | 0,99584<br>9  | 0,9908<br>20  | 0,01480<br>8 | 0,1809<br>69 | 1,00336<br>8  | 1,0027<br>06  | 0,88081<br>7 | 0,8735<br>57 | 0,86005<br>6 | 0,8613<br>35 | 0,85101<br>5 | 0,8591<br>18 |

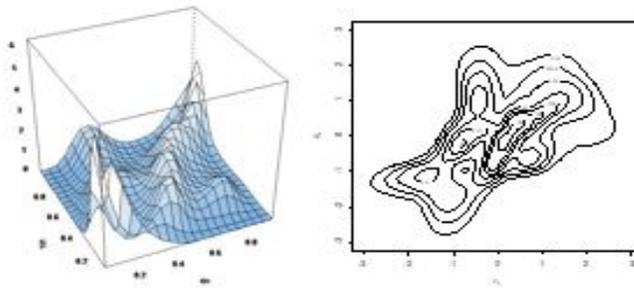
|            |          |           |          |           |          |           |          |           |          |           |          |           |          |           |
|------------|----------|-----------|----------|-----------|----------|-----------|----------|-----------|----------|-----------|----------|-----------|----------|-----------|
| AIC        | 3,525219 | 3,792430  | 5,164512 | 5,187237  | 6,760446 | 6,851179  | 8,034770 | 8,056426  | 6,872058 | 6,898813  | 6,189902 | 6,295171  | 7,200875 | 7,274595  |
| SIC        | 3,489558 | 3,750826  | 5,128850 | 5,145632  | 6,724785 | 6,809574  | 7,999109 | 8,014821  | 6,836397 | 6,857208  | 6,154241 | 6,253566  | 7,165214 | 7,232990  |
| ARCH LM    | 0,058553 | 0,026835  | 14,38796 | 12,3258   | 0,055843 | 0,479833  | 1,530258 | 1,744576  | 0,075620 | 0,001059  | 0,312276 | 0,252751  | 5,438830 | 4,043979  |
| CGARCH     | (0,8088) | (0,8699)  | (0,0001) | (0,0004)  | (0,8132) | (0,4885)  | (0,2161) | (0,1866)  | (0,7833) | (0,9740)  | (0,5763) | (0,6151)  | (0,0197) | (0,0443)  |
| $\theta_0$ | Gaussian | Student-t | Gaussian | Student-t | Gaussian | Student-t | Gaussian | Student-t | Gaussian | Student-t | Gaussian | Student-t | Gaussian | Student-t |
| $\alpha$   | 0,004264 | 0,133065  | 0,002380 | -4,51E-05 | 3,26E-05 | 5,68E-05  | 8,00E-06 | 1,05E-05  | 6,94E-05 | 8,87E-05  | 0,000287 | 0,000225  | 5,78E-05 | 5,69E-05  |
| $\gamma$   | 0,988999 | 0,999862  | 1,000452 | 0,998023  | 0,998160 | 0,994592  | 0,999264 | 0,998944  | 0,967021 | 0,970545  | 0,981147 | 0,995165  | 0,956209 | 0,995410  |
| $\rho$     | 0,124096 | 0,141657  | 0,001481 | 0,000669  | 0,015656 | 0,023294  | 0,006933 | 0,006338  | 0,094871 | 0,130628  | 0,161968 | 0,039984  | 0,135687 | 0,047074  |
| $\delta$   | 0,017968 | 0,039989  | 0,086450 | 0,079463  | 0,061114 | 0,063617  | 0,015811 | 0,023477  | 0,120803 | 0,101371  | 0,056161 | 0,146572  | 0,137520 | 0,172977  |
| $\lambda$  | 0,159894 | 0,090071  | 0,641391 | 0,493453  | 1,43147  | 2,08452   | 0,237465 | 0,004551  | 0,397153 | 0,573178  | 0,624196 | 0,662904  | 0,187214 | 0,654011  |
| AIC        | 3,521265 | 3,763519  | 5,169556 | 5,190938  | 6,827846 | 6,891138  | 8,043947 | 8,061613  | 6,828821 | 6,866843  | 6,103420 | 6,236246  | 7,145757 | 7,250115  |
| SIC        | 3,485603 | 3,721914  | 5,133894 | 5,149333  | 6,792185 | 6,849533  | 8,008286 | 8,020008  | 6,793160 | 6,825238  | 6,067759 | 6,194641  | 7,110096 | 7,208510  |
| ARCH LM    | 0,006679 | 0,179555  | 0,141444 | 0,297775  | 1,516585 | 2,503808  | 0,026993 | 0,007439  | 7,67E-05 | 0,069180  | 1,072496 | 0,05433   | 0,448850 | 1,422144  |
|            | (0,9343) | (0,6718)  | (0,7068) | (0,5853)  | (0,2181) | (0,1136)  | (0,8695) | (0,9313)  | (0,9930) | (0,7925)  | (0,3004) | (0,9412)  | (0,5029) | (0,2331)  |

## Results for the copula models

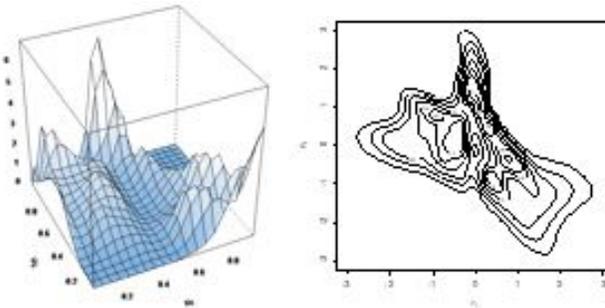
The empirical distribution functions used in modelling the dependence of BTC-US10-Year Bond Yield, BTC-Gold Spot, BTC-US Dollar Index, BTC-FTSE 100, BTC-NIKKEI 225, BTC-S&P 500 pairs are as shown in figure 9, 10, 11, 12, 13 and 14, respectively. We used Clayton, Gumbel Frank, Joe, Gaussian, Student-t, BB8, Survival BB8 and Rotated Tawn Type BB8 270 Degrees copula family. In table 5, it is observed that the relationship between BTC and US Dollar Index is negative, the relationship between BTC and US10 Year Bond Yield, Gold Spot is weak in the positive direction, and the relationship between BTC and FTSE100, Nikkei 225, S&P500 is in the strong positive direction. From table 5, it is clear that the BB8, Survival BB8, Frank and Rotated Tawn Type BB8 270 Degrees copula performs best for the pairs BTC- US10-Year Bond Yield, BTC-Nikkei 225, BTC-Gold Spot, BTC-FTSE 100, BTC-S&P 500 and BTC- US Dollar Index, according to the AIC, and BIC criteria, respectively. In table 5, the calculated tail dependence values for the pairs BTC- US10-Year Bond Yield, BTC-Nikkei 225, BTC-Gold Spot, BTC-FTSE 100, BTC-S&P 500 and BTC- US Dollar Index, when  $\lambda_l = 0$ ,  $\lambda_u = 0$ , symmetric tail dependency is observed in the tail of these pairs. The graphical representations of BTC and used pairs with their three and two dimensional empirical distribution functions are given in figures 10-14, while Clayton, Gumbel Frank, Joe, Gaussian, Student-t, BB8, Survival BB8 and Rotated Tawn Type BB8 270 Degrees copula scatter graphs are shown in figures 15-20, respectively.



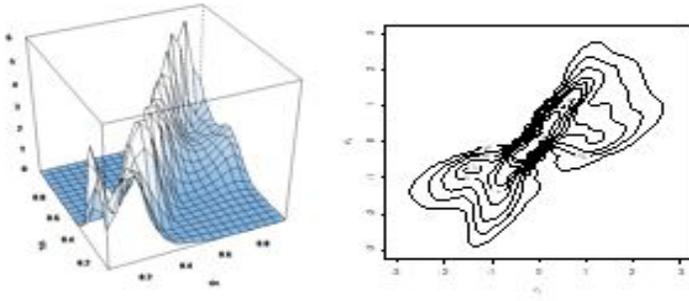
**Figure 9.** For BTC-US10-Year Bond Yield pair three and two dimensional empirical distribution function, respectively



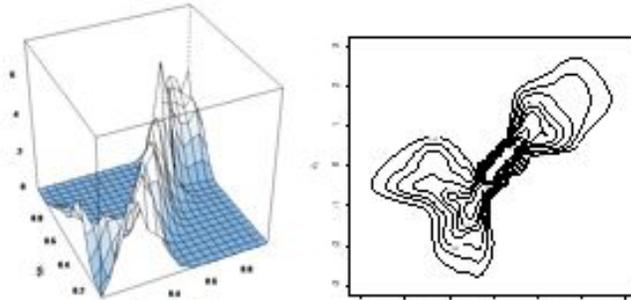
**Figure 10.** For BTC-Gold Spot pair three and two dimensional empirical distribution function, respectively



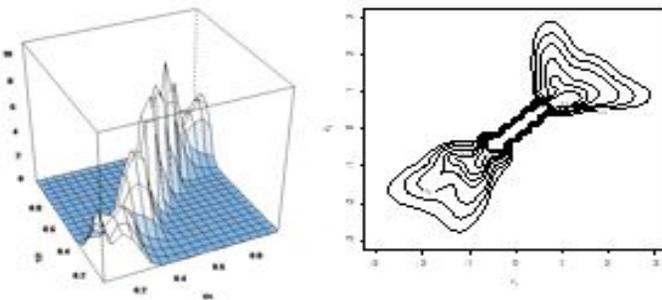
**Figure 11.** For BTC-US Dollar Index pair three and two dimensional empirical distribution function, respectively



**Figure 12.** For BTC-FTSE 100 pair three and two dimensional empirical distribution function, respectively



**Figure 13.** For BTC-NIKKEI 225pair three and two dimensional empirical distribution function, respectively

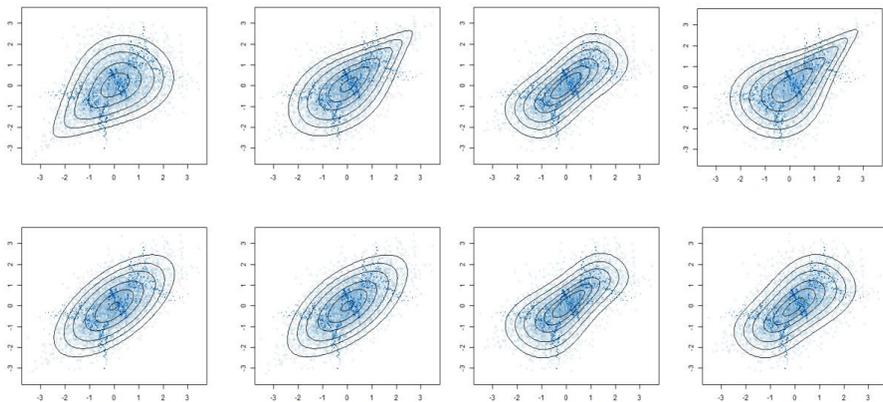


**Figure 14.** For BTC- S&P 500 pair three and two dimensional empirical distribution function, respectively

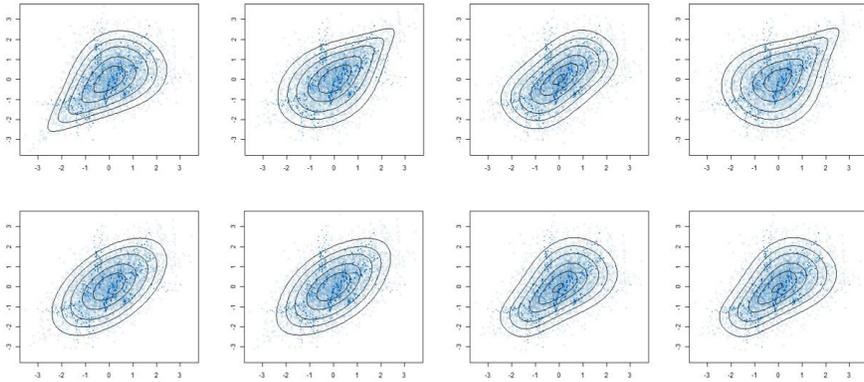
**Table 5.** Estimates for the copula models

|                                     | BTC-<br>US10-<br>Year<br>Bond<br>Yield | BTC-<br>Gold<br>Spot | BTC-<br>US<br>Dollar<br>Index | BTC-<br>FTSE 100 | BTC-<br>NIKKEI<br>225 | BTC-<br>S&P 500 |
|-------------------------------------|----------------------------------------|----------------------|-------------------------------|------------------|-----------------------|-----------------|
| $\tau$                              | 0,47                                   | 0,36                 | -0,36                         | 0,66             | 0,61                  | 0,8             |
| <b>Clayton <math>\theta</math></b>  | 0,7                                    | 0,89                 | -                             | 1,6              | 0,83                  | 2,58            |
| <b>LogL</b>                         | 80,54                                  | 134,81               | -                             | 266,16           | 103,09                | 415,21          |
| <b>AIC</b>                          | -159,09                                | -                    | -                             | -530,31          | -204,18               | -828,43         |
| <b>BIC</b>                          | -154,02                                | -                    | -                             | -525,65          | -199,52               | -823,76         |
| $\lambda_u$                         | 0                                      | 0                    | -                             | 0                | 0                     | 0               |
| $\lambda_l$                         | 0,37                                   | 0,46                 | -                             | 0,65             | 0,43                  | 0,76            |
| <b>Gumbel <math>\theta</math></b>   | 1,7                                    | 1,49                 | -                             | 2,13             | 2,22                  | 2,72            |
| <b>LogL</b>                         | 178,9                                  | 101,99               | -                             | 305,84           | 345,35                | 440,14          |
| <b>AIC</b>                          | -355,8                                 | -                    | -                             | -609,69          | -688,71               | -878,28         |
| <b>BIC</b>                          | -351,13                                | -                    | -                             | -605,02          | -684,04               | -873,61         |
| $\lambda_u$                         | 0,5                                    | 0,41                 | -                             | 0,61             | 0,63                  | 0,71            |
| $\lambda_l$                         | 0                                      | 0                    | -                             | 0                | 0                     | 0               |
| <b>Frank <math>\theta</math></b>    | 5,6                                    | 3,93                 | -3,73                         | 9,64             | 8,36                  | 17,81           |
| <b>LogL</b>                         | 252,12                                 | 136,77               | 133,83                        | 467,66           | 383,85                | 801,45          |
| <b>AIC</b>                          | -502,23                                | -                    | -265,66                       | -933,32          | -765,71               | -1600,9         |
| <b>BIC</b>                          | -497,57                                | -                    | -261                          | -928,65          | -761,04               | -1596,23        |
| $\lambda_u$                         | 0                                      | 0                    | 0                             | 0                | 0                     | 0               |
| $\lambda_l$                         | 0                                      | 0                    | 0                             | 0                | 0                     | 0               |
| <b>Joe <math>\theta</math></b>      | 2,02                                   | 1,53                 | -                             | 2,3              | 3,01                  | 2,84            |
| <b>LogL</b>                         | 157,88                                 | 56,76                | -                             | 209,44           | 359,58                | 290,21          |
| <b>AIC</b>                          | -313,77                                | -                    | -                             | -416,88          | -717,16               | -578,42         |
| <b>BIC</b>                          | -309,1                                 | -                    | -                             | -412,22          | -712,5                | -573,76         |
| $\lambda_u$                         | 0,59                                   | 0,42                 | -                             | 0,65             | 0,74                  | 0,72            |
| $\lambda_l$                         | 0                                      | 0                    | -                             | 0                | 0                     | 0               |
| <b>Gaussian <math>\theta</math></b> | 0,63                                   | 0,54                 | -0,51                         | 0,78             | 0,72                  | 0,86            |
| <b>LogL</b>                         | 191,59                                 | 131,73               | 118,17                        | 368,88           | 284,15                | 512,61          |
| <b>AIC</b>                          | -381,18                                | -                    | -234,33                       | -735,7           | -566,31               | -1023,22        |
| <b>BIC</b>                          | -376,51                                | -                    | -229,66                       | -731,09          | -561,64               | -1018,55        |
| $\lambda_u$                         | 0                                      | 0                    | 0                             | 0                | 0                     | 0               |
| $\lambda_l$                         | 0                                      | 0                    | 0                             | 0                | 0                     | 0               |
| <b>Student <math>t_\nu</math></b>   | 0,62                                   | 0,54                 | -0,51                         | 0,78             | 0,74                  | 0,88            |
| $\rho$                              | 30                                     | 24,23                | 30                            | 30               | 12,75                 | 7,3             |
| <b>LogL</b>                         | 185,66                                 | 132,52               | 114,5                         | 365,68           | 288,19                | 524,76          |

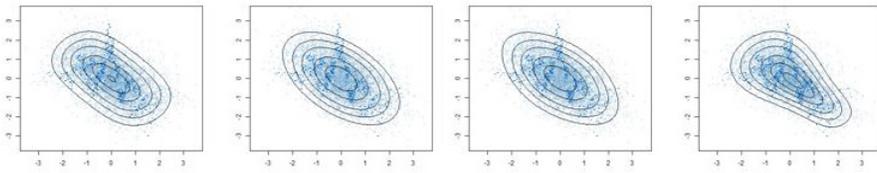
|                                                  |         |        |         |         |         |          |
|--------------------------------------------------|---------|--------|---------|---------|---------|----------|
| AIC                                              | -367,32 | -      | -225,01 | -727,36 | -572,39 | -1045,53 |
| BIC                                              | -357,99 | -      | -215,67 | -718,03 | -563,05 | -1036,19 |
| $\lambda_u$                                      | 0,01    | 0,01   | 0       | 0,06    | 0,17    | 0,48     |
| $\lambda_y$                                      | 0,01    | 0,01   | 0       | 0,06    | 0,17    | 0,48     |
| BB8 $\theta$                                     | 6       | 6      | -       | 6       | 6       | 6        |
| $\sigma$                                         | 0,66    | 0,49   | -       | 0,74    | 0,84    | 0,83     |
| LogL                                             | 269,16  | 124,23 | -       | 387,18  | 460,48  | 566,31   |
| AIC                                              | -534,33 | -      | -       | -770,36 | -916,97 | -1128,63 |
| BIC                                              | -524,99 | -      | -       | -761,03 | -907,63 | -1119,29 |
| $\lambda_u$                                      | 0       | 0      | -       | 0       | 0       | 0        |
| $\lambda_y$                                      | 0       | 0      | -       | 0       | 0       | 0        |
| Survival BB8 $\theta$                            | 6       | 2,83   | -       | 6       | 6       | 6        |
| $\sigma$                                         | 0,57    | 0,86   | -       | 0,8     | 0,64    | 0,88     |
| LogL                                             | 211,49  | 156,17 | -       | 446,65  | 282,44  | 641,81   |
| AIC                                              | -419,49 | -      | -       | -889,3  | -560,89 | -1279,62 |
| BIC                                              | -410,16 | -299   | -       | -879,97 | -551,56 | -1270,28 |
| $\lambda_u$                                      | 0       | 0      | -       | 0       | 0       | 0        |
| $\lambda_y$                                      | 0       | 0      | -       | 0       | 0       | 0        |
| Rotated Taux<br>Type BB8 270<br>degrees $\theta$ | -       | -      | -2,71   | -       | -       | -        |
| $\sigma$                                         | -       | -      | -0,93   | -       | -       | -        |
| LogL                                             | -       | -      | 201,65  | -       | -       | -        |
| AIC                                              | -       | -      | -399,31 | -       | -       | -        |
| BIC                                              | -       | -      | -389,98 | -       | -       | -        |
| $\lambda_u$                                      | -       | -      | 0       | -       | -       | -        |
| $\lambda_y$                                      | -       | -      | 0       | -       | -       | -        |



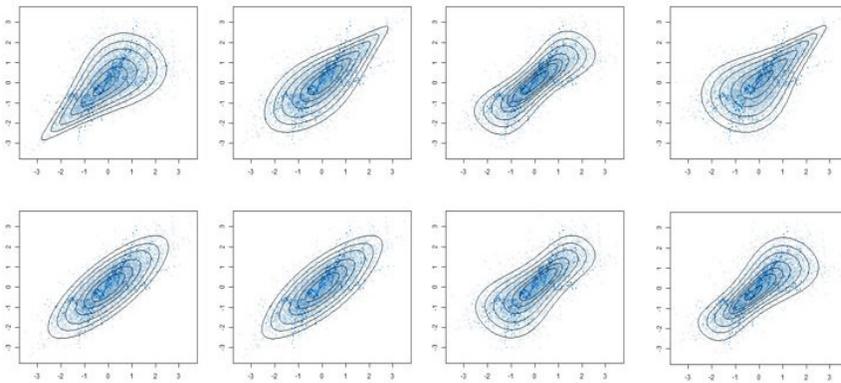
**Figure 15.** For BTC-US10-Year Bond Yield pairs Clayton, Gumbel, Frank, Joe, Gaussian, Student-t, BB8 and Survival BB8 copula scatter graph, respectively.



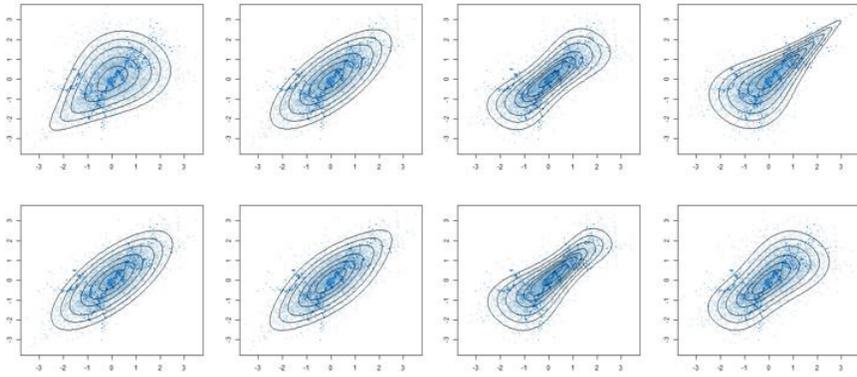
**Figure 16.** For BTC-Gold Spot pairs Clayton, Gumbel, Frank, Joe, Gaussian, Student-t, BB8 and Survival BB8 copula scatter graph, respectively.



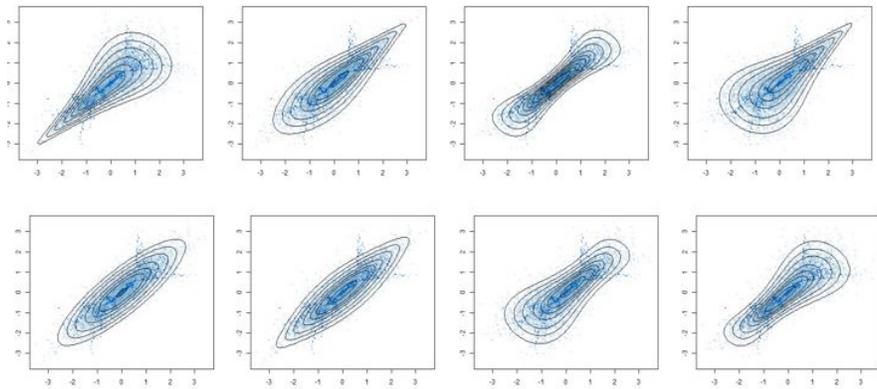
**Figure 17.** For BTC-US Dollar Index pairs Frank, Gaussian, Student-t and Rotated Tawn Type BB8 270 degrees copula scatter graph, respectively.



**Figure 18.** For BTC-FTSE 100 pairs Clayton, Gumbel, Frank, Joe, Gaussian, Student-t, BB8 and Survival BB8 copula scatter graph, respectively.



**Figure 19.** For BTC-NIKKEI 225 pairs Clayton, Gumbel, Frank, Joe, Gaussian, Student-t, BB8 and Survival BB8 copula scatter graph, respectively.



**Figure 20.** For BTC-S&P 500 pairs Clayton, Gumbel, Frank, Joe, Gaussian, Student-t, BB8 and Survival BB8 copula scatter graph, respectively.

## Interpretation of Findings

In this study, Copula-Garch model was used to measure the relationship between Bitcoin and various preeminent indicators. Firstly, Bitcoin and various preeminent indicators were modelled by CGARCH, GJR- GARCH and PGARCH models, which take into account asymmetric effect. It can therefore be clearly said that negative conditions are more effective than positive conditions on serial volatility. In the next stage, the relationship between Bitcoin and various preeminent indicators were modelled by copula, a nonparametric method. We showed that the relationship between BTC and various preeminent indicators is negative, weak positive, and strong positive (Table 5). BTC- US10-Year Bond Yield, BTC-Nikkei 225 pairs were modelled by BB8 copula, and BTC-FTSE 100, BTC-S&P 500 pairs were modelled by Frank copula, BTC-Gold Spot pairs were modelled by Survival BB8 (180 Degrees) copula, and BTC-US Dollar Index pairs were modelled by Rotated Tawn Type BB8 270 Degrees copula. The tails of these pairs show that the Frank copula has zero tail dependence, therefore, BTC-Gold Spot pairs have symmetric tail dependence. The BTC-FTSE 100, BTC-NIKKEI 225, BTC-S&P 500 pairs have upper tail dependency, and BTC-S&P 500 pair has greater upper tail dependency than BTC-FTSE 100 and BTC-NIKKEI 225. Closer linear relationships were found between BTC-FTSE 100, BTC-NIKKEI 225, BTC-S&P 500 when compared to BTC- US10-Year Bond Yield, BTC-Gold Spot, BTC-US Dollar Index.

## Conclusions

Bitcoin, created by a person or group under the pseudonym Nakamoto (2008), in 2009, reached its maximum price on 17 December 2017, at US\$19.780. It is now traded in over 8000 markets, and by 03 January 2018, its total market value surpassed 180 billion dollars. Increasing market share, increasing price and high volatility make Bitcoin appealing for individual users, investors and economists alike. Our analysis supports the findings of Baek and Elbeck (2015) that there is no strong dependence between Bitcoin and other financial indicators. It was observed that Bitcoin's relationship with the Gold Index was much weaker than with other indicators (table 5), supporting the view that Bitcoin is generally regarded as currency rather than an investment tool.

National regulations on Bitcoin differ widely across countries. For example, it is prohibited in Bolivia, and its use is officially restricted in China. In contrast, in Israel, it is subject to the same taxation rules as the local currency and Venezuela has started to initiate a cryptocurrency with the aim

of completely replacing the traditional currency. Institutions such as the IMF, World Bank and the central banks were conceived of to exert economic control through traditional forms of money. However, expected improvements in cryptocurrency systems, and their increasing use globally in the near future will allow the general public to play a more active role in the economic system. Many investment institutions currently avoid cryptocurrencies, but others are in the process of investing in the cryptocurrency business; Goldman Sachs is setting up trading centre for cryptocurrencies, while Chicago Board of Exchange is running Bitcoin Futures. Nevertheless, it should also be noted that, after the entrance of CBOE into Bitcoin Future Market, the Bitcoin price reached a peak, but started to fall dramatically as soon as expectations were fulfilled (Hale et. al., 2018).

In the current situation, it would seem irrational to use Bitcoin as a hedging instrument due to its highly volatile nature. Nevertheless, leading players in the international financial markets are beginning to seriously consider Bitcoin and other cryptocurrencies as a portfolio item and a device to decrease transaction costs. However, its future role is unclear, and will depend on both its movements, and also on the attitude and approaches of governments.

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