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Continuous Dependence on Data for a Solution of Higher Order Quasi-Linear Parabolic Equation Using Fourier Method

İrem BAĞLAN[1*] and Timur CANEL[2]

[1] *Department of Mathematics, Kocaeli University, Kocaeli 41380, Turkey,* isakinc@kocaeli.edu.tr

[2] *Department of Mathematics, Kocaeli University, Kocaeli 41380, Turkey* tcanel@kocaeli.edu.tr

Abstract

Higher order coefficient inverse parabolic quasi-linear problem was examined for this article .In the article, Fourier and explicit finite difference methods are used,respectively. It showed continuously dependence data (stability) of the solution,using iteration method.

Keywords: Inverse coefficient problem; higher order problem; periodic boundary conditions.

1. *Introduction*

Inverse problems are used to find an unknown property of material. For example, scattered by an acoustic plane wave, the far dispersed field can be observed by observing. It can find the shape and material by (inverse) problem such problems flying objects, especially airplanes, missiles, submarines, etc. It is also important in the definition of inverse problems underground sends out a geophysical acoustic wave. like this collects on the surface of the earth and scattered space.It helps to find inhomogeneities and irregularities. Frequency is very important in technology, for example, a hole in a metal.There may be a tumor or some abnormalities in medicine.If inhomogeneities can be found in an environment by processing the scattering area on surface, then you don't have to drill a hole middle. Higher order inverse parabolic problems especially used in chemical diffusion applications, (Sharma, Methi 2012), (Cannon 1989), (Dehghan 2001,2003,2005). Heat transfer processes such as population, medical field, electrochemistry, engineering, chemistry, plasma physics, mathematics. (Ergün 2019),(Amirov,Ergün 2020).

In this work, Fourier method is studied for the solution of common problem:

$$\frac{\partial v}{\partial t} = b(t)\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + g(x,y,t,v),$$

$$v(x,y,0) = \varphi(x,y),$$

$$v(0,y,t) = v(\pi,y,t),$$
$$v(x,0,t) = v(x,\pi,t),$$

$$v_x(0,y,t) = v_x(\pi,y,t),$$
$$v_y(x,0,t) = v_y(x,\pi,t),$$

$$k(t) = \int_0^{\pi}\int_0^{\pi} xyv(x,y,t)dxdy, t \in [0,T],$$

where $x \in [0,\pi], y \in [0,\pi], t \in [0,T]$ and $\varphi(x,y)$, $g(x,y,t,v)$ unknown functions and $k(t)$ is total quantity of heat (Ionkin 1977).

2. *Analysis of Solution of (1)-(4) Model*

As known, in Fourier Method, the solution of problem is considered in the following form :

$$v(x,y,t) = \frac{v_0(t)}{4}$$
$$+ \sum_{m,n=1}^{\infty} v_{scmn}(t)\sin(2mx)\cos(2ny)$$
$$+ \sum_{m,n=1}^{\infty} v_{smn}(t)\sin(2mx)\sin(2ny).$$

We have Fourier coefficients by applying the Fourier method, as follows:

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Corresponding author: İrem BAĞLAN, PhD
Kocaeli University, Faculty of Science and Lettters, Department of Mathematics.
E-mail: isakinc@kocaeli.edu.tr
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$$v_0(t) = v_0(0) + \frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi g(x,y,\tau,v)\,dx\,dy\,d\tau$$

$$v_{cmn}(t) = v_{cmn}(0)e^{-\int_0^t\left[b(s)(2m)^2+(2n)^2\right]ds}$$
$$+\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi e^{-\int_\tau^t\left[b(s)(2m)^2+(2n)^2\right]ds} g\cos(2mx)\cos(2ny)\,dx\,dy\,d\tau$$

$$v_{csmn}(t) = v_{csmn}(0)e^{-\int_0^t\left[b(s)(2m)^2+(2n)^2\right]ds}$$
$$+\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi e^{-\int_\tau^t\left[b(s)(2m)^2+(2n)^2\right]ds} g\cos(2mx)\sin(2ny)\,dx\,dy\,d\tau$$

$$v_{scmn}(t) = v_{scmn}(0)e^{-\int_0^t\left[b(s)(2m)^2+(2n)^2\right]ds}$$
$$+\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi e^{-\int_\tau^t\left[b(s)(2m)^2+(2n)^2\right]ds} g\sin(2mx)\cos(2ny)\,dx\,dy\,d\tau$$

$$v_{smn}(t) = v_{smn}(0)e^{-\int_0^t\left[b(s)(2m)^2+(2n)^2\right]ds}$$
$$+\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi e^{-\int_\tau^t\left[b(s)(2m)^2+(2n)^2\right]ds} g\sin(2mx)\sin(2ny)\,dx\,dy\,d\tau$$

Then we obtain the solution:

$$v(x,y,t) = \frac{1}{4}\left(\varphi_0 + \frac{4}{\pi^2}\int_0^t g_0(\tau,v)\,d\tau\right)$$
$$+\sum_{m,n=1}^{\infty}\left(\varphi_{cmn} + \frac{4}{\pi^2}\int_0^t e^{-\int_\tau^t\left[b(s)(2m)^2+(2n)^2\right]ds} g_{cmn}(\tau,v)\,d\tau\right)\cos(2mx)\cos(2ny)$$
$$+\sum_{m,n=1}^{\infty}\left(\varphi_{csmn} + \frac{4}{\pi^2}\int_0^t e^{-\int_\tau^t\left[b(s)(2m)^2+(2n)^2\right]ds} g_{csmn}(\tau,v)\,d\tau\right)\cos(2mx)\sin(2ny)$$
$$+\sum_{m,n=1}^{\infty}\left(\varphi_{scmn} + \frac{4}{\pi^2}\int_0^t e^{-\int_\tau^t\left[b(s)(2m)^2+(2n)^2\right]ds} g_{scmn}(\tau,v)\,d\tau\right)\sin(2mx)\cos(2ny)$$

where

$$\varphi_0 = v_0(0),$$

$$\varphi_{cmn} = v_{cmn}(0)e^{-\int_0^t\left[b(s)(2m)^2+(2n)^2\right]ds},$$

$$\varphi_{csmn} = v_{csmn}(0)e^{-\int_0^t\left[b(s)(2m)^2+(2n)^2\right]ds},$$

$$\varphi_{scmn} = v_{scmn}(0)e^{-\int_0^t\left[b(s)(2m)^2+(2n)^2\right]ds},$$

$$\varphi_{smn} = v_{smn}(0)e^{-\int_0^t\left[b(s)(2m)^2+(2n)^2\right]ds}.$$

We have the following constraints for functions of the problem:

(S1) $k(t) \in C^1[0,T]$

(S2) $\varphi(x,y)\varepsilon C^{1,1}([0,\pi]\times[0,\pi])$,

$$\int_0^\pi\int_0^\pi xy\varphi(x,y)dxdy = k(0),$$

(S3)For $g(x,y,t,v)$

$$\left|\frac{\partial g(x,y,t,v)}{\partial x} - \frac{\partial g(x,y,t,\bar{v})}{\partial x}\right| \le l(x,y,t)\left|v-\bar{v}\right|,$$

$$\left|\frac{\partial g(x,y,t,v)}{\partial y} - \frac{\partial g(x,y,t,\bar{v})}{\partial y}\right| \le l(x,y,t)\left|v-\bar{v}\right|,$$

$$\left|\frac{\partial g(x,y,t,v)}{\partial x\partial y} - \frac{\partial g(x,y,t,\bar{v})}{\partial x\partial y}\right| \le l(x,y,t)\left|v-\bar{v}\right|,$$

where $l(x,y,t) \in L_2(D), l(x,y,t) \ge 0$,

(2) $g(x,y,t,v) \in C^{2,2,0}[0,\pi],\ t \in 0,T]$,

(3) $g(x,y,t,v)\big|_{x=0} = g(x,y,t,v)\big|_{x=\pi}$,

Let provided upper conditions for x,y,xy partial derivative.

(5) k(t) can be diffrentiated under the assumptions (C1)-(C3),

$$\int_0^\pi\int_0^\pi xyv_t(x,t)dxdy = k'(t), 0 \le t \le T.$$

then the unknown coefficient is obtained in this form

$$b(t) = \frac{k'(t) - \int_0^\pi\int_0^\pi xyg(x,y,t,v)dxdy - \frac{\pi^3}{2}v_y(\pi,t)}{\frac{\pi^3}{2}v_x(\pi,t)}.$$

Definition 2.1.

$$\{v(t)\} = \{v_0(t), v_{cmn}(t), v_{csmn}(t), v_{scmn}(t), v_{smn}(t)\}$$

of continuous functions on [0, T] which satisfy the condition

$$\max_{0\le t\le T}\frac{|v_0(t)|}{4} + \sum_{m,n=1}^{\infty}\left(\begin{array}{l}\max\limits_{0\le t\le T}|v_{cmn}(t)| + \max\limits_{0\le t\le T}|v_{csmn}(t)| \\ + \max\limits_{0\le t\le T}|v_{scmn}(t)| + \max\limits_{0\le t\le T}|v_{smn}(t)|\end{array}\right) < \infty.$$

$$\|v(t)\| = \max_{0\le t\le T}\frac{|v_0(t)|}{4} + \sum_{m,n=1}^{\infty}\left(\begin{array}{l}\max\limits_{0\le t\le T}|v_{cmn}(t)| + \max\limits_{0\le t\le T}|v_{csmn}(t)| \\ + \max\limits_{0\le t\le T}|v_{scmn}(t)| + \max\limits_{0\le t\le T}|v_{smn}(t)|\end{array}\right)$$

is the norm in B. (B is the Banach spaces).

3. *Stability of the Solution*

Theorem 3.1 According to (S1)-(S3) the solution is constantly dependent on the given data.

Proof. Supposed $\theta = \{\varphi, k, g\}$ and $\bar{\theta} = \{\bar{\varphi}, \bar{k}, \bar{g}\}$, M, L_i, $i = 1,2$ are positive constants such that

$$\|g\|_{C^{1,1,0}[\Gamma]} \le M, \|\bar{g}\|_{C^{1,1,0}[\Gamma]} \le M,$$

$$\|\varphi\|_{C^3[0,\pi]} \le L_1, \|\bar{\varphi}\|_{C^3[0,\pi]} \le L_1,$$

$$\|k\|_{C^1[0,T]} \le L_2, \|\bar{k}\|_{C^1[0,T]} \le L_2,$$

Let we take

$$\|\theta\| = (\|k\|_{C^1[0,T]} + \|\varphi\|_{C^{1,1}[0,\pi]} + \|g\|_{C^{1,1,0}[\bar{\Gamma}]}).$$

Let (b,u) and $(\bar{b},\bar{u})$ be solutions :

$$v - \bar{v} = \frac{(\varphi_0 - \overline{\varphi_0})}{4} + \sum_{m,n=1}^{\infty}\varphi_{cmn}e^{-\int_\tau^t\left[b(s)(2m)^2+(2n)^2\right]ds}\cos(2mx)\cos(2ny)$$

$$\sum_{m,n=1}^{\infty}\overline{\varphi_{cmn}}e^{-\int_{\tau}^{t}\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\cos(2mx)\cos(2ny)$$

$$+\sum_{m,n=1}^{\infty}\varphi_{csmn}e^{-\int_{\tau}^{t}\left[b(s)(2m)^2+(2n)^2\right]ds}\cos(2mx)\sin(2ny)$$

$$+\sum_{m,n=1}^{\infty}\overline{\varphi_{csmn}}e^{-\int_{\tau}^{t}\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\cos(2mx)\sin(2ny)$$

$$+\sum_{m,n=1}^{\infty}\varphi_{scmn}e^{-\int_{\tau}^{t}\left[b(s)(2m)^2+(2n)^2\right]ds}\sin(2mx)\cos(2ny)$$

$$+\sum_{m,n=1}^{\infty}\overline{\varphi_{scmn}}e^{-\int_{\tau}^{t}\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\sin(2mx)\cos(2ny)$$

$$+\sum_{m,n=1}^{\infty}\varphi_{smn}e^{-\int_{\tau}^{t}\left[b(s)(2m)^2+(2n)^2\right]ds}\sin(2mx)\sin(2ny)$$

$$+\sum_{m,n=1}^{\infty}\overline{\varphi_{smn}}e^{-\int_{\tau}^{t}\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\sin(2mx)\sin(2ny)$$

$$+\frac{1}{4}\left(\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi\left[g(x,y,\tau,v)-g(x,y,\tau,\bar{v})\right]e^{-\int_{\tau}^{t}\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}dxdyd\tau\right)$$

$$+\frac{1}{4}\left(\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi g(x,y,\tau,\bar{v})\left(e^{-\int_{\tau}^{t}\left[b(s)(2m)^2+(2n)^2\right]ds}-e^{-\int_{\tau}^{t}\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\right)dxdyd\tau\right)$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi\left[g(x,y,\tau,v)-g(x,y,\tau,\bar{v})\right]e^{-\int_{\tau}^{t}\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\cos(2mx)\cos(2ny)dxdyd\tau$$

$$+\sum_{k=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi g(x,y,\tau,\bar{v})\left(e^{-\int_{\tau}^{t}\left[b(s)(2m)^2+(2n)^2\right]ds}-e^{-\int_{\tau}^{t}\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\right)\cos(2mx)\cos(2ny)dxdyd\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi\left[g(x,y,\tau,v)-g(x,y,\tau,\bar{v})\right]e^{-\int_{\tau}^{t}\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\cos(2mx)\sin(2ny)dxdyd\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi g(x,y,\tau,\bar{v})\left(e^{-\int_\tau^t\left[b(s)(2m)^2+(2n)^2\right]ds}-e^{-\int_\tau^t\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\right)\cos(2mx)\sin(2ny)\,dxdyd\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi\left[g(x,y,\tau,v)-g(x,y,\tau,\bar{v})\right]e^{-\int_\tau^t\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\sin(2mx)\cos(2ny)\,dxdyd\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi g(x,y,\tau,\bar{v})\left(e^{-\int_\tau^t\left[b(s)(2m)^2+(2n)^2\right]ds}-e^{-\int_\tau^t\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\right)\sin(2mx)\cos(2ny)\,dxdyd\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi\left[g(x,y,\tau,v)-g(x,y,\tau,\bar{v})\right]e^{-\int_\tau^t\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\sin(2mx)\sin(2ny)\,dxdyd\tau$$

$$+\sum_{m,n=1}^{\infty}\frac{4}{\pi^2}\int_0^t\int_0^\pi\int_0^\pi g(x,y,\tau,\bar{v})\left(e^{-\int_\tau^t\left[b(s)(2m)^2+(2n)^2\right]ds}-e^{-\int_\tau^t\left[\bar{b}(s)(2m)^2+(2n)^2\right]ds}\right)\sin(2mx)\sin(2ny)\,dxdyd\tau$$

We applying Cauchy,Hölder,Bessel,Lipschitz inequalities, we obtain,

$$\|v-\bar{v}\|_B\le\frac{\|\varphi_0-\overline{\varphi_0}\|}{4}$$

$$+\sum_{m,n=1}^{\infty}\|\varphi_{cmn}-\overline{\varphi_{cmn}}\|+\|\varphi_{csmn}-\overline{\varphi_{csmn}}\|$$
$$+\|\varphi_{scmn}-\overline{\varphi_{scmn}}\|+\|\varphi_{smn}-\overline{\varphi_{smn}}\|$$
$$+\sqrt{T}\left(\frac{3\sqrt{\pi}+16}{3\pi}\right)\|l(x,y,t)\|_{L_2(\Gamma)}\|v(t)-\bar{v}(t)\|_B$$
$$+\sqrt{T}\left(\frac{3\sqrt{\pi}+16}{3\pi}\right)\|l(x,y,t)\|_{L_2(\Gamma)}M\|b(t)-\bar{b}(t)\|$$

where

$$\|\theta-\bar{\theta}\|=\frac{\|\varphi_0-\overline{\varphi_0}\|}{4}$$
$$+\sum_{m,n=1}^{\infty}\|\varphi_{cmn}-\overline{\varphi_{cmn}}\|+\|\varphi_{csmn}-\overline{\varphi_{csmn}}\|$$
$$+\|\varphi_{scmn}-\overline{\varphi_{scmn}}\|+\|\varphi_{smn}-\overline{\varphi_{smn}}\|.$$

$$\|b(t)-\bar{b}(t)\|_{C[0,T]}\le C\|v(t)-\bar{v}(t)\|_B$$

where

$$C=\left(\frac{\pi M}{2\|u(t)\|_B\|\bar{u}(t)\|_B}+\frac{\pi\|l(x,y,t)\|_{L_2(\Gamma)}}{2\|u(t)\|_B}\right).$$

$$\|v-\bar{v}\|_B^2\le 2D^2\|\theta-\bar{\theta}\|^2$$

where

$$D=\frac{1}{1-\left(\sqrt{T}\left(\frac{3\sqrt{\pi}+16}{3\pi}\right)\|l(x,y,t)\|_{L_2(\Gamma)}+MC\right)}.$$

For $\theta\to\bar{\theta}$ then $v\to\bar{v}$. Hence $b\to\bar{b}$.

4. *The Numerical Examination*

If we linearize, we take:

$$\omega_t^{(n)} = b(t)\omega_{xx}^{(n)} + \omega_{yy}^{(n)} + g(x,y,t,\omega^{(n-1)}),$$

$$\omega^{(n)}(x,y,0) = \varphi(x,y),$$

$$\omega^{(n)}(0,y,t) = \omega^{(n)}(\pi,y,t),$$
$$\omega^{(n)}(x,0,t) = \omega^{(n)}(x,\pi,t),$$

$$\omega_x^{(n)}(0,y,t) = \omega_x^{(n)}(\pi,y,t),$$

$$\omega_y^{(n)}(x,0,t) = \omega_y^{(n)}(x,\pi,t), x \in [0,\pi], t \in [0,T].$$

If we take $\omega^{(n)}(x,y,t) = \nu(x,y,t)$ and $g(x,y,t,\omega^{(n-1)}) = \overline{g}(x,y,t)$.

Then we have a linear problem:

$$\nu_t = b(t)\nu_{xx} + \nu_{yy} + \overline{g}(x,y,t), \quad (x,y,t) \in \Gamma$$

$$\nu(x,y,0) = \varphi(x,y), x \in [0,\pi], y \in [0,\pi]$$

$$\nu(0,y,t) = \nu(\pi,y,t), y \in [0,\pi], t \in [0,T]$$

$$\nu(x,0,t) = \nu(x,\pi,t), x \in [0,\pi], t \in [0,T]$$

$$\nu_x(0,y,t) = \nu_x(\pi,y,t), y \in [0,\pi], t \in [0,T]$$
$$\nu_y(x,0,t) = \nu_y(x,\pi,t), y \in [0,\pi], t \in [0,T]$$

$[0,\pi]^2 \times [0,T]$ is divided to an $M^2 \times N$ mesh with the step sizes $h = \pi/M$, $\tau = T/N$. Let's take $v_{i,j}^k$, $g_{i,j}^k$, φ_i and b^k that instead of $v(x_i, y_j, t_k)$, $g(x_i, y_j, t_k)$, $\varphi(x_i, y_j)$ and $b(t_k)$.

According to implicit finite-difference method for the last problem :

$$\frac{1}{\tau}\left(v_{i,j}^{k+1} - v_{i,j}^{k}\right) = \frac{1}{h^2}\begin{bmatrix} b^k\left(v_{i-1,j}^{k+1} - 2v_{i,j}^{k+1} + v_{i+1,j}^{k+1}\right) \\ + \left(v_{i,j-1}^{k+1} - 2v_{i,j}^{k+1} + v_{i,j+1}^{k+1}\right)\end{bmatrix} + g_{i,j}^{k+1}, \quad 3)$$

$$v_{i,j}^0 = \varphi_i, \quad 4)$$

$$v_{0,j}^k = v_{M+1,j}^k, v_{M+1,j}^k = \frac{v_{1,j}^k - v_{M,j}^k}{2} \quad 5)$$

$$v_{i,0}^k = v_{i,M+1}^k, v_{i,M+1}^k = \frac{v_{i,1}^k - v_{i,M}^k}{2}$$

$$b(t) = \frac{k'(t) - \int_0^\pi\int_0^\pi xy\, g(x,y,t)dxdy - \frac{\pi^3}{2}v_y(\pi,t)}{\frac{\pi^3}{2}v_x(\pi,t)}. \quad (16)$$

$$b^{k+1} = -\frac{\left(\left(k^{k+2} - k^k\right)/\tau\right)}{\left(\frac{\pi^3}{2}v_x(\pi,t)\right)^k} - \frac{\left(\int_0^\pi\int_0^\pi xy\, g(x,y,t)dxdy\right)^k}{\left(\frac{\pi^3}{2}v_x(\pi,t)\right)^k} - \left(\frac{\pi^3}{2}v_y(\pi,t)\right)^k \quad (11)$$

where $k^k = k(t_k)$, $k = 0,1,...,N$.

According to the integration rule applying Simpson's central difference scheme then

$b^{k(s)}$, $v_{i,j}^{k(s)}$ are the values of b^{k}, $v_{i,j}^{k}$ at te s-th iteration step. $b^{k+1(s+1)}$ is find same estimations.

$$\frac{1}{\tau}\left(v_{i,j}^{k+1(s+1)}-v_{i,j}^{k+1(s)}\right)$$
$$=\frac{1}{h^{2}}\begin{bmatrix} b^{k(s+1)}\left(v_{i-1,j}^{k+1(s+1)}-2v_{i,j}^{k+1(s+1)}+v_{i+1,j}^{k+1(s+1)}\right) \\ +\left(v_{i,j-1}^{k+1(s+1)}-2v_{i,j}^{k+1(s+1)}+v_{i,j+1}^{k+1(s+1)}\right) \end{bmatrix}$$
$$+g_{i,j}^{k+1},$$

$$v_{i,j}^{0}=\varphi_{i},$$

$$v_{0,j}^{k+1(s)}=v_{M+1,j}^{k+1(s)},v_{M+1,j}^{k+1(s)}=\frac{v_{1,j}^{k+1(s)}-v_{M,j}^{k+1(s)}}{2}$$

$$v_{i,0}^{k+1(s)}=v_{i,M+1}^{k+1(s)},v_{i,M+1}^{k+1(s)}=\frac{v_{i,1}^{k+1(s)}-v_{i,M}^{k+1(s)}}{2}$$

$v_{i,j}^{k+1(s+1)}$ is found.

5. *Discussion*

The inverse time-dependent coefficient for two-dimensional nonlinear parabolic equation with periodic and integral conditions has examined. This problem has been examined by two part. Fistly, theoretical part of the study, stability of the problem have been showed. Secondly, for numerical part, The iteration are showed. Especially periodic boundary conditions is the use of these conditions. Nonlokal (this conditions) are very hard than local boundary conditions with this kind of problem. In this study, the Fourier and finite difference methods were used for this problem. The authors are considering dealing with other inverse coefficients problems in future studies.

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